6.3 Progression

The progression of the sequence (1), 2, 3, 4, ..., n, denoted as (n), follows a specific pattern.

For the progression, we can define the sequence as follows:

\[ a_n = a_1 + (n-1)d \]

where:
- \( a_n \) is the nth term of the sequence,
- \( a_1 \) is the first term of the sequence,
- \( d \) is the common difference,
- \( n \) is the term number.

The progression can be visualized as a linear function where each term increases by a constant value, \( d \), from one term to the next.

6.4 Quadratic Progression

A quadratic progression is a sequence where the difference between consecutive terms is not constant but increases linearly. This results in a parabolic pattern.

For a quadratic progression, the nth term can be represented as:

\[ a_n = a_1 + (n-1)d + \frac{1}{2}(n-1)(n-2)c \]

where:
- \( c \) is the common second difference.

This formula allows us to calculate any term in the sequence given the first term, the common difference, and the common second difference.
\[ f = \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} \sin(nx) \]

Recall that the Fourier series expansion of a function \( f \) in the interval \( [-\pi, \pi] \) is given by:

\[ f(x) = \sum_{n=1}^\infty \frac{a_n}{2} + \sum_{n=1}^\infty a_n \cos(nx) + \sum_{n=1}^\infty b_n \sin(nx) \]

where the coefficients \( a_n \) and \( b_n \) are calculated as follows:

\[ a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) \, dx \]
\[ b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) \, dx \]

For the given function \( f(x) = \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} \sin(nx) \), we have:

\[ a_n = 0 \quad \text{and} \quad b_n = \begin{cases} \frac{(-1)^{n+1}}{n} & \text{if } n \text{ is odd} \\ 0 & \text{otherwise} \end{cases} \]

Therefore, the Fourier series expansion of \( f(x) \) is:

\[ f(x) = \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} \sin(nx) \]

This series converges to \( f(x) \) at every point in the interval \([-\pi, \pi]\). The partial sums of the series approximate the function \( f(x) \) as the number of terms increases.

The graph of the function and its Fourier series expansion is shown below, illustrating the convergence of the series to the original function at most points.
where

\[ 0 \leq \frac{n}{n^2 - n + 1} \leq 1 \]

and

\[ \frac{\ln(n) - \ln(n+1)}{n^2 - n + 1} \leq 0 \]

for all \( n \geq 1 \).

Moreover, if \( \lim_{n \to \infty} \frac{n}{n^2 - n + 1} = 0 \), then the sequence converges to 0.

Finally, for any \( n \geq 1 \), we have

\[ \frac{n}{n^2 - n + 1} \leq \frac{1}{n} \leq 1 \]

which implies

\[ \frac{1}{n^2 - n + 1} \leq \frac{1}{n} \leq 1 \]

and

\[ \lim_{n \to \infty} \frac{1}{n} = 0 \]

therefore

\[ \frac{1}{n^2 - n + 1} \leq 0 \]

which means

\[ \frac{n}{n^2 - n + 1} \leq 1 \]

and

\[ \frac{\ln(n) - \ln(n+1)}{n^2 - n + 1} \leq 0 \]

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Thus, the sequence converges to 0.